

Neutrino Flux Bounds and Prospects for High Energy and Ultrahigh Energy Neutrino Source Detection

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Abstract. After briefly reviewing various hadronic neutrino source models, we show how to construct generic upper flux bounds. We then turn to the problem of neutrino propagation through the inner Earth and neutrino detection in water-based Čerenkov detectors. Applying the formalism thus developed to the Mannheim-Protheroe-Rachen (MPR) and the Waxman & Bahcall flux bounds, we find that event rates of several hundred to thousand events per year might be possible in next-generation neutrino telescopes. However, a tomography of the inner Earth will face severe constraints due to the statistical error of the event rates to be expected.

1 Hadronic neutrino source models

Models for sources producing ultrahigh energy neutrinos can be divided into hadronic and non-hadronic ones, depending on whether protons play a key part in the neutrino production. In this work we shall limit ourselves to the former class of models, referring the reader to Learned and Mannheim (2000) for a discussion of non-hadronic source models such as the decay of relic particles.

Independent of the specific source under consideration, the main idea behind hadronic neutrino production is that a beam of accelerated protons (taken to be of power-law form, $n_p(E_p) \propto E_p^{-s}$) hitting a target produces pions, which subsequently decay and thus yield neutrinos. Here, the target may consist of either protons or photons.

In case of a target made of protons, if the pions decay prior to any interaction with the surrounding medium, the resulting neutrino production spectrum is of the same form as the original spectrum of accelerated protons, i.e. $Q_\nu^{pp}(E_\nu) \propto E_\nu^{-s}$. Otherwise, the neutrino spectrum is steepened (Rachen and Mészáros, 1998).

An important example of a photon target is given by the cosmic background radiation, which leads to the Greisen-Zatsepin-Kuzmín cutoff. In neutrino sources, target photons

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may be produced by means of synchrotron radiation. Assuming that the synchrotron photon spectrum is of the form $N_\gamma(E_\gamma) \propto E_\gamma^{-\alpha}$, one may show that the corresponding neutrino spectrum is also a power law, $Q_\nu^{p\gamma}(E_\nu) \propto E_\nu^{-(s-\alpha)}$ (Learned and Mannheim, 2000).

In the following, we mention several examples of source models which have been proposed for ultrahigh energy neutrino production:

Terrestrial atmosphere:

Neutrinos produced by cosmic-ray interactions in the terrestrial atmosphere can be detected with present-day water-based Čerenkov telescopes and thus serve as an important benchmark (Domogatsky, 2001; Andres et al., 2001).

Sun:

The same processes responsible for neutrino production in the terrestrial atmosphere can also take place in the Sun. However, the neutrino flux thus to be expected is so low that it will only be discernible with next-generation neutrino detectors (Moskalenko et al., 1991; Hettlage et al., 2000).

Crab nebula:

Neutrons resulting from the photodisintegration of iron nuclei may be accelerated in the outer gap of the Crab pulsar and may subsequently decay into protons. If these interact with matter in the nebula, pions and thus neutrinos can be produced (Bednarek and Protheroe, 1997).

Supernova remnants:

Protons may be shock accelerated and by means of collisions in the remnant may produce pions and thus neutrinos (Gaisser et al., 1998).

AGNs:

In active galactic nuclei (AGNs), protons may be shock accelerated either in the accretion disk of the central black hole or in the jets. Depending on the site of acceleration, either the thermal photons of the accretion disk or the synchrotron radiation in the jet may serve as the target needed for pion production (e.g. Nellen et al. (1993); Stecker and Salamon (1996)).

It should be noted that the detection of neutrinos emitted by AGNs would be a smoking gun for hadron acceleration.

Galaxy clusters:

The ionized gas in a galaxy cluster might act as the target for cosmic rays escaping from cluster galaxies (Colafrancesco and Blasi, 1998).

GRBs:

Protons might be accelerated in gamma-ray bursts (GRBs) (e.g. Waxman and Bahcall (1997, 2000); De Paolis et al. (2001)). The main advantage of gamma-ray bursts is that they allow a precise timing, which reduces the background considerably. In addition, assuming that two neutrinos emitted at the same moment in a GRB located at a distance d arrive at different times t and $t + \Delta t$, one may compute the speed difference Δv_ν of the two neutrinos by means of

$$\frac{\Delta v_\nu}{c} \approx 10^{-18} \left(\frac{d}{10\text{Gpc}} \right)^{-1} \left(\frac{\Delta t}{1\text{s}} \right).$$

Hence GRBs could offer a possibility for checking neutrino dispersion.

Fig. 1 shows the muon neutrino fluxes predicted for various source models, if no neutrino flavor oscillations are assumed.

2 Neutrino flux bounds

The multitude (and uncertainty) of neutrino source models suggests the question whether one can obtain a generic upper neutrino flux bound, thus constraining the number of events to be expected from future neutrino telescopes. In order to construct such a bound, we start by assuming that in a neutrino source all protons are confined within the acceleration region (while neutrons and neutrinos may escape), that there is a power law target photon spectrum with index α , $n_\gamma(E_\gamma) \propto E_\gamma^{-\alpha}$, and that the upper limit on the cosmic ray proton spectrum as observed on Earth has the form $n_{p,\text{obs}}(E_p) \propto E_p^{-2.75}$ between 3×10^6 and 10^{12} GeV (Mannheim et al., 2001).

Particle physics considerations then show that the neutrino production rate Q_{ν_μ} within the source is given by

$$Q_{\nu_\mu}(E_{\nu_\mu}) \approx \begin{cases} 83.3 Q_n(25E_{\nu_\mu}) & (\text{if } \alpha = 1) \\ 416 Q_n(25E_{\nu_\mu}) & (\text{if } \alpha = 0) \end{cases}, \quad (1)$$

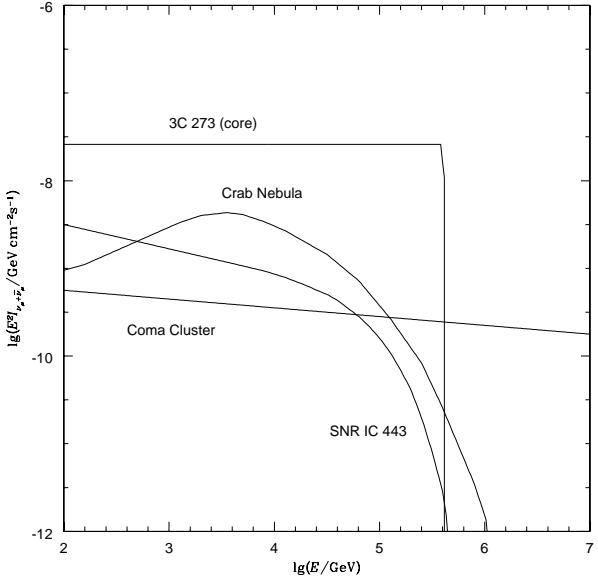


Fig. 1. Muon neutrino and antineutrino fluxes for 3C 273 (Nellen et al., 1993; Learned and Mannheim, 2000), the Crab nebula (Bednarek and Protheroe, 1997), the supernova remnant SNR IC 443 (Gaisser et al., 1998; Learned and Mannheim, 2000), and the Coma cluster (Colafrancesco and Blasi, 1998). No neutrino flavor oscillations are assumed.

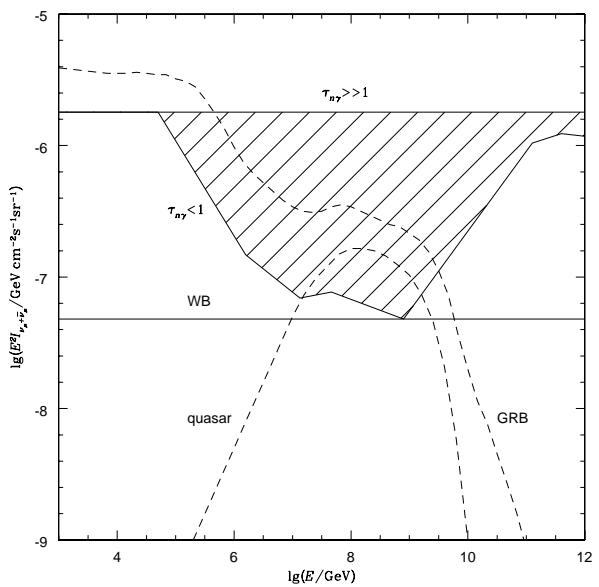


Fig. 2. Generic muon neutrino and antineutrino flux bound for sources with low ($\tau_{n\gamma} < 1$) and high neutron opacity ($\tau_{n\gamma} \gg 1$) (MPR flux bounds), and generic bound for neutrinos emitted by quasars (quasar), as obtained by Mannheim et al. (2001). The shaded region is the allowed range for the MPR flux bounds. Also shown are the generic GRB flux bound given by Mannheim (2001) (GRB) and the flux bound obtained by Waxman and Bahcall (1999) (WB). No neutrino flavor oscillations are assumed.

where Q_n denotes the neutron production rate (Mannheim et al., 2001). The source contributes to the cosmic ray proton flux, if the neutron escape probability $P_{\text{esc},n}$ does not vanish. Indeed, the cosmic ray production rate can be estimated as (Mannheim et al., 2001)

$$\begin{aligned} Q_{\text{cr}}(E_p) &\approx Q_n(E_p) \times P_{\text{esc},n}(E_p) \approx \frac{1 - e^{-\tau}}{\tau} Q_n(E_p) \\ &\approx Q_n(E_p) \times \begin{cases} 1 & (\tau < 1) \\ \tau^{-1} & (\tau > 1) \end{cases}. \end{aligned} \quad (2)$$

Here τ constitutes the neutron optical depth for the source radius. Finally, we note that the bolometric photon and neutrino luminosities are related by $L_\gamma = 2L_\nu$ for $\alpha = 1$ and $L_\gamma \approx L_\nu$ for $\alpha = 0$ (Rachen and Mészáros, 1998).

We can now give a formal recipe for constructing generic upper flux bounds (Mannheim et al., 2001):

1. Get the neutron production rate Q_n . Note that due to the rescaling in step 4, Q_n need to be known up to a factor of proportionality only.
2. By means of Eqs. 1 and 2 compute the corresponding neutrino and cosmic ray proton production rates (i.e. Q_{ν_μ} and Q_{cr}).
3. Integrate the neutrino and cosmic ray proton production rates over all sources in the universe to obtain the neutrino and cosmic ray proton intensities I at the Earth. Formally, this may be done using the formula

$$I(E) \propto \frac{1}{4\pi} \int_{z_{\min}}^{z_{\max}} M(E, z) \frac{(1+z)^2}{4\pi d_L^2} \frac{dV_c}{dz} \frac{dP_{\text{source}}}{dV_c} \cdot Q((1+z)E, z) dz,$$

where z denotes the redshift, d_L the luminosity distance, V_c the comoving volume, and dP_{source}/dV_c the source density, which analogously to Q_n in step 1 needs to be known only up to a factor of proportionality. The modification factor M takes into account the attenuation processes along the line of flight. For neutrinos, $M(E, z) = 1$. The factor $1 + z$ in the argument of Q describes the energy degradation due to the expansion of the universe.

4. Normalize I_{cr} and thus I_{ν_μ} so that I_{cr} is tangent to the observational upper limit of the cosmic ray proton intensity or so that γ -rays aren't overproduced (whichever is the more stringent condition).

It should be noted that the bounds thus computed pertain to hadronic source models only, so that there might be an additional contribution from non-hadronic sources. One may now use this recipe for computing various bounds:

Generic bounds (Mannheim et al., 2001) are obtained, if one assumes that the neutron production spectrum is given by $Q_n(E_n) \propto E_n^{-1} \exp(-E_n/E_{\max})$ with a cutoff energy E_{\max} and that the source distribution follows that of galaxies and AGNs. The corresponding cosmic ray proton flux

as a function of E_{\max} is fitted to the observational bound. We shall refer to these bounds as the Mannheim-Protheroe-Rachen (MPR) flux bounds.

In order to get a bound for EGRET blazars (Mannheim et al., 2001), we assume that the production rate of accelerated protons is of the form $Q_p(E_p) \propto E_p^{-2} \exp(-E_p/E_{\max})$, corresponding to non-relativistic Fermi acceleration, and that the spectral index of the target photon spectrum has the value $\alpha = 1$. From Q_p and the proton loss time scale one may compute the proton spectrum N_p in the source, and from N_p and the timescale of $p\gamma \rightarrow n$ the neutron production rate Q_n , which may serve as the starting point for the general recipe outlined above. Here, as the production rates depend on the luminosity, the average over the luminosity function dN/dL is used:

$$\langle Q_{\nu_\mu, \text{cr}}(E) \rangle = \frac{\int Q_{\nu_\mu, \text{cr}}(E, L) \frac{dN}{dL} dL}{\int \frac{dN}{dL} dL}.$$

For GRBs (Mannheim, 2001) one takes the spectrum of accelerated protons to be of the form $Q_p(E_p) \propto E_p^{-2}$, but assumes that the photon target spectral index is given by $\alpha = 0$. The calculation can then be carried out analogously to that for the blazar bound.

Finally, if it is assumed that the proton input spectrum is of the form $Q_p(E_p) \propto E_p^{-2}$ and that the sources are optically thin for neutrons, one obtains the Waxman & Bahcall bound (Waxman and Bahcall, 1999).

In Fig. 2 various upper flux bounds for the muon neutrino flux from hadronic sources are shown.

3 Event rates in neutrino telescopes

The decay of a charged pion leads to the production of one electron and two muon neutrinos, where here and in the following no distinction between particles and antiparticles is made. This implies that initially the ratio of the various neutrinos should be given by $Q_{\nu_e} : Q_{\nu_\mu} : Q_{\nu_\tau} = 1 : 2 : 0$.

However, during the flight to Earth neutrino flavor oscillations should occur, leading to an equipartition among the various flavors (Learned and Pakvasa, 1995; Husain, 2000). Hence the flux ratio as measured on Earth should have the value

$$I_{\nu_e} : I_{\nu_\mu} : I_{\nu_\tau} = 1 : 1 : 1,$$

so that a considerable amount of tauon neutrinos is to be expected, although virtually no tauon neutrinos are produced within neutrino sources. Neutrinos reaching a detector from below have to cross the inner Earth first. This propagation can formally be described by means of the set (Hettlage and Mannheim, 2001)

$$\frac{dI_{\nu_i}}{dt} = -\sigma_{i,\text{tot}} I_{\nu_i} + \sum_k \int_E^\infty \frac{d\sigma_{k \rightarrow i}(E', E)}{dE} I_{\nu_k}(E'), \quad (3)$$

of cascade equations, where σ_i denotes the total neutrino cross section of flavor i , the sum runs over all flavors, and

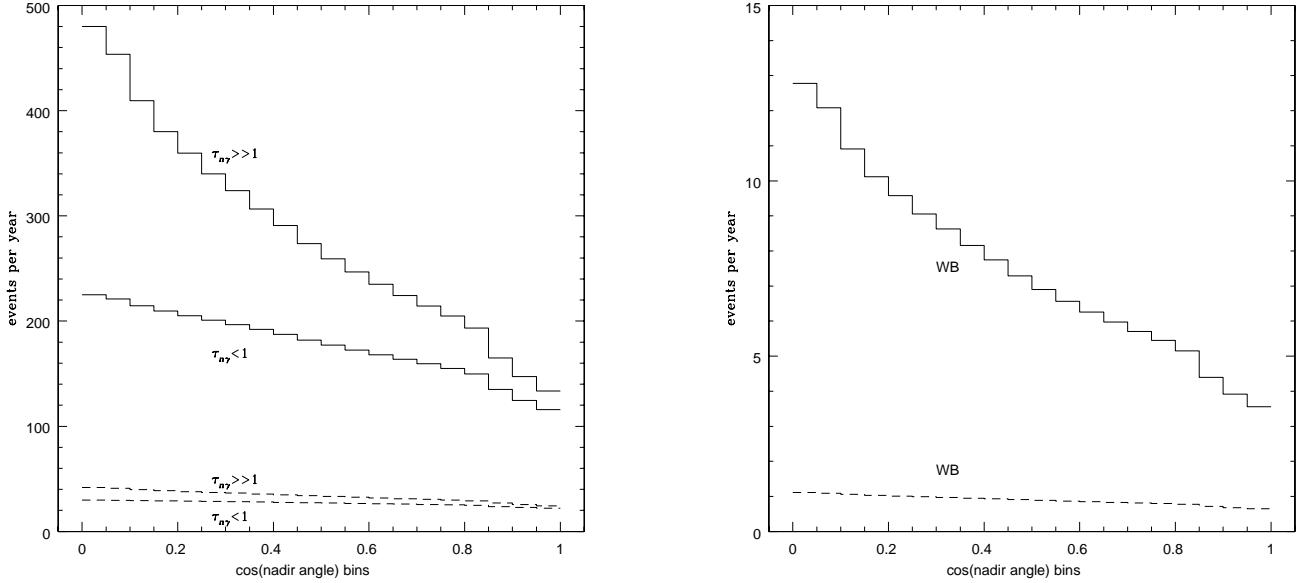


Fig. 3. Event rates of muon (*solid*) and tauon neutrinos (*dashed lines*) for bins of the cosine of the nadir angle. *Left:* Event rates for the MPR flux bounds with high ($\tau_{n\gamma} \gg 1$) and low neutron source opacity ($\tau_{n\gamma} < 1$). *Right:* Event rates for the Waxman & Bahcall flux bound.

$d\sigma_{k \rightarrow i}/dE$ covers both neutrino-nucleon and leptonic neutrino interactions, and the decay of tauons. For the parton distributions we use the CTEQ5DIS functions (Lai, 2000), and we discretize the transport equations. The resulting set of equations can then be solved semi-analytically.

As a rough approximation to the event rate \dot{N} due to neutrino detection in water-based Čerenkov telescopes, we employ the formula

$$\begin{aligned} \dot{N}_i(E) \approx & \frac{\rho A_{\text{eff}}}{m_p} \left(\int_E^\infty dE' \frac{d\sigma_i^{\text{CC}}}{dE'} I_i(E') L_{\text{det}} \right. \\ & + \left. \int_L dx \int_E^\infty dE' \frac{d\sigma_i^{\text{CC}}}{dE'} (E_0^{(i)}(E', x)) I_i(E_0^{(i)}(E', x)) \right). \quad (4) \end{aligned}$$

Here again i denotes the flavor. A_{eff} constitutes the effective detector area, L the detector height, ρ the matter density, and m_p the proton mass. $E_0^{(i)}(E', x)$ is the energy (of the lepton i) which is degraded to E' along a path of length x . In the following, we shall assume $A_{\text{eff}} = 1 \text{ km}^2$, $L = 1 \text{ km}$, and $\rho = 1 \text{ g/cm}^3$.

Whereas the first term on the right hand side of Eq. 4 describes fully contained events, the second term covers leptons originating outside the detector. It may be neglected for electrons and tauons.

Finally, it ought to be noted that the decay of tauons results in the production of both electrons and muons and hence contributes to the respective event rates.

4 Results

The formalism outlined in the previous section can be used to obtain the event rates corresponding to various of the flux

bounds discussed in Sect. 2. This has been carried out for the Waxman & Bahcall and the MPR flux bounds. The results (for a lower energy cutoff of 100 GeV) are given in Fig. 3.

As in Fig. 3 different bins correspond to the same solid angle, the form of the curves is a direct measure of attenuation effects due to crossing the Earth. This suggests that one might use the event rate as a function of the nadir angle in order to perform a tomography of the inner Earth (Jain et al., 1999).

To this end, we start by noting that the column density X may be considered as the one-dimensional Radon transform of the number density ρ :

$$X(p, \xi) = \int \rho(r) \delta(p - \xi r) d^2r. \quad (5)$$

Here, p and ξ are the distance from the middle of the Earth and the normal vector of the line of integration, respectively. Now, using the inversion formula for two-dimensional Radon transforms (Deans, 1993) one can show that $X(p, \xi) \equiv X(p)$ and that one may thus invert Eq. 5 by means of

$$\rho(r) = -\frac{1}{2\pi r} \frac{d}{dr} \int_{-\sqrt{R^2-r^2}}^{+\sqrt{R^2-r^2}} X(\sqrt{r^2+q^2}) dq, \quad (6)$$

where $\rho(r) \equiv 0$ for $r > R$. However, if the initial neutrino spectrum (before crossing the Earth) is assumed to be known, one may deduce the column density directly from the observed event rate. Then Eq. 6 yields the density, so that a tomography is indeed possible.

Unfortunately, there is an important caveat: Due to the statistical error σ in the detection rates, the column densities can be known only with a rather limited precision. We take this into account by either adding or subtracting σ throughout and comparing the results thus obtained with that for $\sigma = 0$.

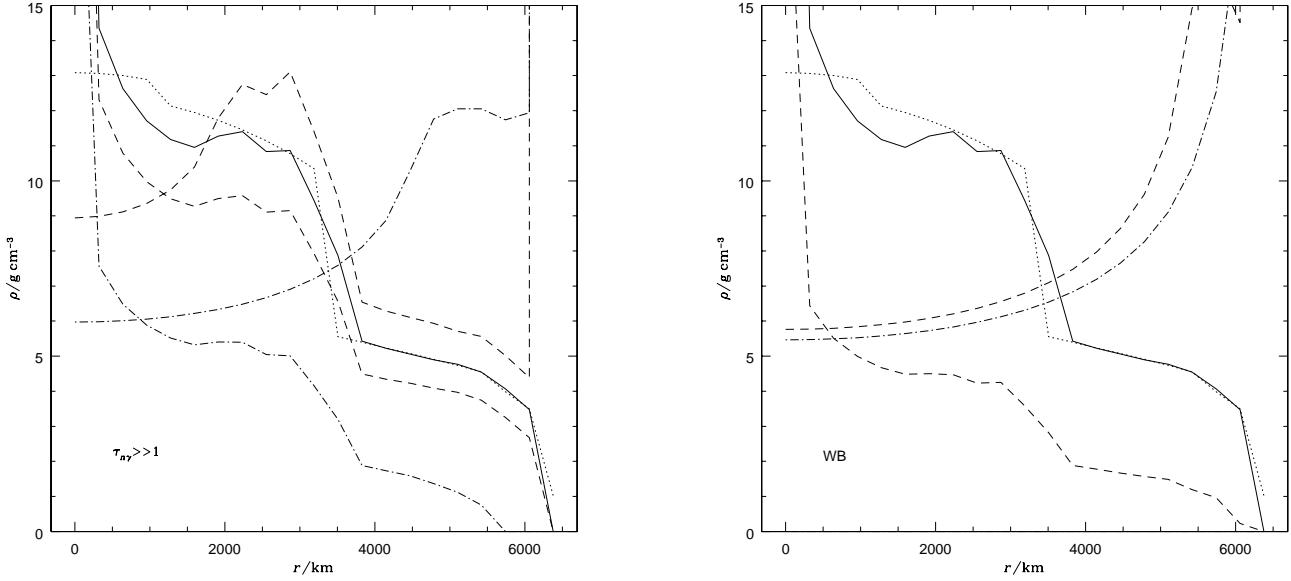


Fig. 4. *Left:* Earth density ρ as a function of the radius r obtained by performing a tomography on the event rates \dot{N} of Fig. 3 for the MPR flux bound with high neutron source opacity. Shown are the results for no statistical error (*solid*), for $\dot{N}_\mu \pm \sqrt{\dot{N}_\mu}$ (*dashed*), and for $\dot{N}_\tau \pm \sqrt{\dot{N}_\tau}$ (*dot-dashed* lines). A lower energy cutoff of 100 GeV is assumed. For comparison the “correct” density according to Dziewonski and Anderson (1981) is included (*dotted* line). *Right:* Same as left figure, but for the Waxman & Bahcall flux bound.

The results of this analysis for a one-year run are shown in Fig. 4 for an MPR and for the Waxman & Bahcall flux bound. One can see that the maximum event rates to be expected for next-generation neutrino telescopes put severe constraints on a tomography of the inner Earth.

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References

Andres, E. et al. (AMANDA Collaboration), Observation of High-Energy Neutrinos Using Cerenkov Detectors Embedded Deep in Antarctic Ice, *Nature*, 410, 441, 2001.

Bednarek, W. and Protheroe, R.J., Gamma Rays and Neutrinos from the Crab Nebula Produced by Pulsar Accelerated Nuclei, *Phys. Rev. Lett.*, 79, 2616, 1997.

Colafrancesco, S. and Blasi, P., Clusters of Galaxies and the Diffuse Gamma Ray Background, *Astropart. Phys.*, 9, 227, 1998.

Deans, S.R., The Radon Transform and Some of its Applications, Krieger Pub. Co, Malabar, 1993.

De Paolis, F. et al., High-Energy Neutrino Emission from Gamma-Ray Bursts, *astro-ph/0107589*.

Domogatsky, G.V., The BAIKAL Neutrino Project: Status Report, *Nucl. Phys. B (Proc. Suppl.)*, 91, 438, 2001.

Dziewonski, A.M. and Anderson, D.L., Preliminary Reference Earth Model, *Phys. Earth Planet. Inter.*, 25, 297, 1981.

Gaisser, T.K. et al., Gamma-Ray Production in Supernova Remnants, *Astrophys. J.*, 492, 219, 1998.

Hettlage, C. and Mannheim, K., Tau Events in Neutrino Telescopes, *Nucl. Phys. B (Proc. Suppl.)*, 95, 165, 2001.

Hettlage, C. et al., The Sun as a High Energy Neutrino Source, *Astropart. Phys.*, 13, 45, 2000.

Husain, A., High Energy Cosmic Tau Neutrinos, *Nucl. Phys. B (Proc. Suppl.)*, 87, 442, 2000.

Jain, P. et al., Neutrino Absorption Tomography of the Earth’s Interior Using Isotropic Ultra-high Energy Flux, *Astropart. Phys.*, 12, 193, 1999.

Lai, H.L., Global QCD Analysis of Parton Structure of the Nucleon: CTEQ5 Parton Distributions, *Eur. Phys. J. C*, 12, 375, 2000.

Learned, J.G. and Mannheim, K., High-Energy Neutrino Astrophysics, *Ann. Rev. Nucl. Part. Sci.*, 50, 679, 2000.

Learned, J.G. and Pakvasa, S., Detecting ν_τ Oscillations at PeV Energies, *Astropart. Phys.*, 3, 267, 1995.

Mannheim, K., Neutrinos from Gamma Ray Bursts. In: Aharonian, F.A. and Völk, H.J. (Eds.), *High Energy Gamma-Ray Astronomy*, AIP Conference Proceedings, 558, 417, 2001.

Mannheim, K. et al., On the Cosmic Ray Bound for Models of Extragalactic Neutrino Production, *Phys. Rev. D*, 63, 023003, 2001.

Moskalenko, I.V. et al., The Sun as a Source of VHE Neutrinos, *Astron. Astrophys.*, 248, L5, 1991.

Nellen, L. et al., Neutrino Production through Hadronic Cascades in AGN Accretion Disks, *Phys. Rev. D*, 47, 5270, 1993.

Rachen, J.P. and Mészáros, P., Photohadronic Neutrinos from Transients in Astrophysical Sources, *Phys. Rev. D*, 58, 123005, 1998.

Stecker, F.W. and Salamon, M.H., High Energy Neutrinos from Quasars, *Space Sci. Rev.*, 75, 341, 1996.

Waxman, E. and Bahcall, J.N., High Energy Neutrinos from Cosmological Gamma-Ray Burst Fireballs, *Phys. Rev. Lett.*, 78, 2292, 1997.

Waxman, E. and Bahcall, J.N., High Energy Neutrinos from Astrophysical Sources: An Upper Bound, *Phys. Rev. D*, 59, 023002, 1999.

Waxman, E. and Bahcall, J.N., Neutrino Afterglow from Gamma-Ray Bursts: $\sim 10^{18}$ eV, *Astrophys. J.*, 541, 707, 2000.